On the Evolution of Primordial Magnetic Fields

Konstantinos Dimopoulos *

and

Anne–Christine Davis †

Department of Applied Mathematics and Theoretical Physics,
University of Cambridge, Silver Street,
Cambridge, CB3 9EW, U.K.

February 5, 2008

Abstract

A new mechanism for the evolution of primordial magnetic fields is described and analysed. The field evolution is followed from the time of its creation until the epoch of structure and galaxy formation. The mechanism takes into account the turbulent behaviour of the early universe plasma, whose properties determine strongly the evolution of the field configuration. A number of other related issues such as the case of an electroweak plasma are also considered. Finally, as an example, the mechanism is applied to specific models.

DAMTP-96-17

*e-mail: K.Dimopoulos@damtp.cam.ac.uk
†e-mail: A.C.Davis@damtp.cam.ac.uk

1. One of the most exciting astrophysical consequences of phase transitions in the early universe is the possible creation of primordial magnetic fields. The existence of a primordial magnetic field could have significant effect on various astrophysical processes. In particular it may be involved in the galaxy formation process [1], [2] or in the generation of the observed galactic magnetic fields.

It is widely accepted that the galactic magnetic fields are generated through a galactic dynamo mechanism. Here, a weak seed field is exponentially amplified by the turbulent motion of ionized gas, which follows the differential rotation of the galaxy [3], [4]. The currently observed magnetic field of the Milky Way and of nearby galaxies is of the order of a $\mu Gauss$. If the e-folding time is no more than the galactic rotation period, $\sim 10^8 yrs$, then, considering the galactic age, $\sim 10^{10} yrs$, the seed field needed to produce a field of the observed value is about $\sim 10^{-19} Gauss$ [3] on a comoving scale of a protogalaxy ($\sim 100\,kpc$). Since the gravitational collapse of the protogalaxies enchances their frozen-in magnetic field by a factor of $(\rho_G/\rho_0)^{2/3} \sim 10^3$, where $\rho_G \sim 10^{-24} g\,cm^{-3}$ is the typical mass density of a galaxy and $\rho_0 \simeq 2 \times 10^{-29} \Omega h^2 g\,cm^{-3}$ is the current cosmic mass density, this seed field corresponds to an rms field of the order of $\sim 10^{-22} Gauss$ over the comoving scale of $\sim 1\,Mpc$. With the assumption that the rms field scales approximately as a^{-2} with the expansion of the universe, where $a \propto t^{2/3}$ is the scale factor in the matter era, we find that the required rms value of the seed field at the time $t_{eq} \sim 10^{11} sec$ of equal matter and radiation densities is $\sim 10^{-22} Gauss \times (t_{gc}/t_{eq})^{4/3} \sim 10^{-20} Gauss$, where $t_{gc} \sim 10^{15} sec$ is the time of the gravitational collapse of the galaxies [1]. Thus, an rms field of magnitude, $B^{eq}_{rms} \sim 10^{-20} Gauss$ at t_{eq} would be sufficient to seed the galactic dynamo and generate the observed galactic magnetic fields.

Various attempts have been made to produce a primordial field in the early universe [5]. In most of the cases, though, the achieved field appeared to be too weak or incoherent to seed the galactic dynamo. In this letter, it is shown that the key issue, determining the rms value of a primordial magnetic field at t_{eq}, is the evolution of the correlated domains of the field, i.e. the growth of the lengthscale over which the magnetic field is coherent. We develop a detailed mechanism for the evolution of the magnetic field configuration and we show that, in contrast to what is usually assumed, the correlation length, in general, grows faster than the scale factor a. This results in more coherent rms fields at the epoch of galaxy formation.

2. In the existing literature the entire magnetic field configuration is taken to be comovingly frozen. As a result the rms magnetic field is thought to evolve as a^{-2}, due to flux conservation.

The fact that the early universe is, with great accuracy, a perfect conductor, ensures that magnetic flux is, indeed, conserved (in absense of dissipation) and, therefore, the magnetic field can be thought to be "frozen" into the plasma [6], [7], over a certain scale. However, the simplistic assumption that the correlated domains of the field expand only due to the Hubble expansion does not take into account that the faster expanding causal correlations, through electromagnetic turbulence, could rearrange the field and correlate it on comoving scales larger than its initial correlated domains. This is because, when two initially uncorrelated neighbouring domains come into causal contact, the magnetic field

around the interface is expected to untangle and smooth, in order to avoid the creation of energetically unfavoured magnetic domain walls. In time the field inside both domains "aligns" itself and becomes coherent over the total volume. The velocity v, with which such a reorientation occurs, is determined by the plasma, which carries the field and has to reorientate its motion for that purpose.[1]

Thus, the evolution of the correlation length is given by,

$$\frac{\mathrm{d}\xi}{\mathrm{d}t} = H\xi + v \tag{1}$$

where ξ is the correlation length of the magnetic field configuration, H is the Hubble parameter and v is the peculiar, bulk velocity, determined, in principle, by the state of the plasma.

From (1) it is apparent that the correlated domains could grow faster than Hubble expansion. Therefore, *the magnetic field configuration is not necessarily comovingly frozen.* Indeed we show that the domains can expand much faster than the universe, resulting in large correlations of the field.

Here it is important to point out our basic implicit assumption, which concerns the damping of the small scale structure of the magnetic field as the correlated domains expand. Indeed, although we will not refer to specific damping mechanisms, *we assume that all the Fourier modes of the magnetic field with wavelength smaller than the dimensions of the correlated domains are fully damped* and, therefore, the field is coherent inside these domains. This is equivalent to assuming that the magnetohydrodynamic backreaction, which could transfer power to the small scales at later times, is not effective. This assumption enables us to avoid the intrinsic non-linearities of the problem (by confining them into the damping mechanisms) and to attempt a linear approach *without using perturbation theory.* Of course, a more realistic picture would have to include some transfer of power to the smaller than the correlation length scales, especially the ones that are near the scale of the correlated domains. In that sense our work can be viewed as *the optimum case*, i.e. the limit of the fastest possible growth of the correlated domains. As such we can still have predictive power by setting upper limits to the strength and coherence of the magnetic field configuration at any given time.

In order to describe the evolution of the correlated domains one has to determine the peculiar velocity v of equation (1). This primarily depends on the opacity of the plasma.

If the plasma is opaque on the scale of a correlated domain, then radiation cannot penetrate this scale and is blocked inside the plasma volume. Consequently, the plasma is subject to the total magnetic pressure of the magnetic field gradient energy. Therefore, this energy dissipates through coherent magnetohydrodynamic oscillations, i.e. Alfven waves. This is evident, since the coherent, bulk velocity of the plasma motion is driven by the magnetic field so that $\rho v^2 \sim B^2$. Thus, there is equipartition of energy between the

[1] Note that the plasma does not have to be carried from one domain to another or get somehow mixed. Also, conservation of flux is not violated with the field's rearrangements, since *the field always remains frozen into the plasma*, which is carried along.

coherent motion of the plasma and the magnetic filed[2]. Consequently, in this case, the peculiar velocity of the magnetic field reorientation, is the well known Alfven velocity [8],[3]

$$v_A \equiv \frac{B_{cd}}{\sqrt{\rho}} \tag{2}$$

where B_{cd} is the magnitude of the magnetic field inside a correlated domain and ρ is the *total* energy density of the universe, since, before t_{eq}, matter and radiation are strongly coupled.[4]

Now, if the plasma is not opaque over the scale ξ of a correlated domain, then radiation can penetrate this scale and carry away momentum, extracted from the plasma through Thomson scattering of the photons. This subtraction of momentum is equivalent to an effective drag force, $F \sim \rho\,\sigma_T\,v_T\,n_e$ [8]. Balancing this force with the magnetic force determines the "Thomson" velocity over the scale ξ,

$$v_T \equiv \frac{v_A^2}{\xi n_e \sigma_T} \tag{3}$$

where v_A is the Alfven velocity, n_e is the electron number density and σ_T is the Thomson cross-section.

Hence, for a non–opaque plasma the peculiar velocity of the plasma reorientation is given by [8],

$$v = \min(v_A, v_T) \tag{4}$$

In order to explore the behaviour of the opaqueness of the plasma, we need to compare the mean free path of the photon $l_T \sim (n_e\sigma_T)^{-1}$ to the scale ξ of the correlated domains. For realistic models, the correlated domains remain opaque at least until the epoch $t_{anh} \sim 0.1\,sec$ of electron–positron annihilation ($T \sim 1\,MeV$). The reason for this can be easily understood by calculating l_T before and after pair annihilation.

For $T > 1\,MeV$, instead of the usual Thomson cross-section σ_T, we have the Klein-Nishina cross-section [9],

$$\sigma_{KN} \simeq \frac{3}{8}\,\sigma_T\,(\frac{m_e}{T})\,[\,\ln\frac{2T}{m_e} + \frac{1}{2}\,] \simeq 2.7\,(\frac{GeV}{T})\,\ln[\frac{T}{GeV}]\,GeV^{-2} \tag{5}$$

where $m_e \simeq 0.5\,GeV$is the electron mass and $\sigma_T \simeq 6.65 \times 10^{-25} cm^2 \simeq 1707.8\,GeV^{-2}$. The electron number density is given by [10],

$$n_e \simeq \frac{3}{4}\frac{\zeta(3)}{\pi^2}\,g_e T^3 \tag{6}$$

[2]The coherent plasma motion should not be confused with the thermal motion where $v_{th} \simeq \sqrt{T/m}$.

[3]Unless explicitly specified, natural units are being used ($\hbar = c = 1$).

[4]This coupling implies that any reorientation of the momentum of matter has to drag radiation along with it. This increases the inertia of the plasma, that balances the magnetic pressure.

where $\zeta(3) \simeq 1.20206$ and $g_e = 4$ are the internal degrees of freedom of electrons and positrons.

From (5) and (6) we find,

$$l_T \sim \frac{0.1\, GeV}{T^2} \qquad \text{for } T > 1\, MeV \tag{7}$$

which at annihilation gives, $l_T(t_{anh}) \sim 10^5 GeV^{-1}$.

After annihilation the electron number density is given by [10],

$$n_e \simeq 6 \times 10^{-10} n_\gamma \simeq 1.44 \times 10^{-10} T^3 \tag{8}$$

where n_γ is the photon number density given by,

$$n_\gamma \simeq \frac{\zeta(3)}{\pi^2}\, g_\gamma T^3 \tag{9}$$

where $g_\gamma = 2$ are internal degrees of freedom of the photon.

With the usual value for σ_T we obtain,

$$l_T \sim \frac{10^6 GeV^2}{T^3} \qquad \text{for } T < 1\, MeV \tag{10}$$

At annihilation the above gives, $l_T(t_{anh}) \sim 10^{15} GeV^{-1}$.

Hence, the mean free path of the photon at the time of pair annihilation is enlarged by a factor of 10^{10}! As a result, l_T is very likely to become larger than ξ after t_{anh}. If this is so, the Thomson dragging effect has to be taken into account and the peculiar velocity of the plasma reorientation is given by (4).

In order to calculate the peculiar velocity it is necessary to compute the Alfven velocity, which requires the knowledge of the magnetic field value B_{cd} inside a correlated domain. To estimate that we assume that the magnetic flux, on scales very much larger than the sizes of the correlated domains, is conserved, as implied by the frozen–in condition.

Consider a closed curve C in space, of lengthscale $L > \xi$, encircling an area A. Conservation of flux suggests that the flux averaged mean magnetic field inside A scales as a^{-2}. This implies that for the field inside a correlated domain we have, $B_{cd}(L/\xi)^{-1} \propto a^{-2}$. Since C follows the universe expansion $L \propto a$, with $a \propto t^{1/2}$. Thus, for the radiation era, we obtain,

$$B_{cd}\, t^{1/2} \xi = K \Rightarrow B_{cd} = \frac{K}{t^{1/2}\xi} \tag{11}$$

where K is a constant to be evaluated at any convenient time. We will show that the correlation length grows at least as fast as the universe expands. This implies that the magnetic field inside a correlated domain dilutes at least as rapidly as a^{-2} for the radiation era.

Subsituting the above into (2) we find,

$$v_A \sim 10\,\frac{K}{m_{pl}}\,\frac{t^{1/2}}{\xi} \tag{12}$$

where we have also used the well known relation,

$$t \simeq 0.3\, g_*^{-1/2} (\frac{m_{pl}}{T^2}) \tag{13}$$

where $m_{pl} \simeq 1.22 \times 10^{19} GeV$ is the Planck mass and g_* is the number of particle degrees of freedom, which, for $T < 1\, MeV$ is 3.36 [10]. [5]

Solving the evolution equation (1) with $a \propto t^{1/2}$ in the case that $v = v_A$ gives,

$$\xi(t)^2 = (\frac{t}{t_i})\, \xi_i^2 + 4 v_A(t)\, \xi(t)\, t\, (1 - \sqrt{\frac{t_i}{t}}\,) \tag{14}$$

where ξ_i is the correlation length of the field at the time t_i. The first term of (14) is due to the Hubble expansion, whereas the second term is due to the peculiar velocity.

In the case of $v = v_T$, for $t > t_{anh}$, using (8), (9) and the usual value of σ_T, (3) gives,

$$v_T = D\, \frac{t^{5/2}}{\xi^3} \tag{15}$$

where

$$D \sim 10^{-57} K^2 GeV^{-3/2} \tag{16}$$

Using (15), the evolution equation (1) gives,

$$\xi(t)^4 = (\frac{t}{t_i})^2\, \xi_i^4 + \frac{8}{3}\, v_T(t)\, \xi^3 t\, [1 - (\frac{t_i}{t})^{3/2}] \tag{17}$$

The evolution of the correlation length of the magnetic field configuration is described initially by the Alfven expansion equation (14) until the moment when $\xi \sim l_T$. From then on the growth of ξ continues according either to (14) or to (17), depending on the relative magnitudes of the velocities v_A and v_T. Using the above, the scale ξ_{eq} of the correlated domains at t_{eq} can be estimated. With a suitable averaging procedure, this this can be used to calculate the rms magnetic field over the protogalactic comoving scale at the time when structure formation begins.

3. An important issue, which should be considered, is the diffusion length of the freezing of the field. Indeed, the assumption that the field is frozen into the plasma corresponds to neglecting the diffusion term of the magnetohydrodynamical induction equation [11],

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \sigma^{-1} \nabla^2 \mathbf{B} \tag{18}$$

where $\mathbf{v}$ is the plasma velocity and σ is the conductivity. In the limit of infinite conductivity the diffusion term of (18) vanishes and the field is frozen into the plasma on all scales.

[5] In natural units $G = m_{pl}^{-2}$.

However, if σ is finite then spatial variations of the magnetic field of lengthscale l will decay in a diffusion time, $\tau \simeq \sigma l^2$ [11]. Thus, the field at a given time t can be considered frozen into the plasma only over the diffusion scale,

$$l_d \sim \sqrt{\frac{t}{\sigma}} \tag{19}$$

If $l_d > \xi$, the magnetic field configuration is expected, in less than a Hubble time, to become smooth on scales smaller than l_d. Thus, it is more realistic to consider a field configuration with coherence length $l_d(t)$ and magnitude of the coherent magnetic field $\overline{B_{cd}}$, where $\overline{B_{cd}} = B_{cd}/N$ is the flux-averaged initial magnetic field over $N \equiv l_d/\xi$ number of domains.

An estimate of the plasma conductivity is necessary to determine the diffusion length. The current density in the plasma is given by, $\mathbf{J} = ne\mathbf{v}$, where n is the number density of the charged particles. The velocity $\mathbf{v}$ acquired by the particles due to the electic field $\mathbf{E}$, can be estimated as $\mathbf{v} \simeq e\mathbf{E}\tau_c/m$, where m is the particle mass and $\tau_c = l_{mfp}/v$ is the timescale of collisions. Since the mean free path of the particles is given by, $l_{mfp} \simeq 1/n\sigma_c$, the current density is, $\mathbf{J} \simeq e^2\mathbf{E}/mv\sigma_c$, where σ_c is the collision cross-section of the plasma particles. Comparing with Ohm's law gives for the conductivity [11], [12],

$$\sigma \simeq \frac{e^2}{mv\sigma_c} = \frac{\omega_p^2}{4\pi\nu_c} \tag{20}$$

where $\omega_p \equiv (\frac{4\pi ne^2}{m})^{1/2}$ is the plasma frequency and $\nu_c = nv\sigma_c$ is the frequency of collisions. The collision cross-section is given by the Coulomb formula [12],

$$\sigma_c \simeq \frac{e^4}{T^2} \ln\Lambda \tag{21}$$

where $\ln\Lambda \simeq \ln(e^{-3}\sqrt{T^3/n})$ is the Coulomb logarithm. Thus, the behaviour of the conductivity depends crucially on the temperature.

For low temperatures, $T < m_e \simeq 1\,MeV$ (i.e. after t_{anh}), the velocity of the electrons is, $v \sim \sqrt{T/m_e}$. Thus, from (20) and (21) the conductivity is given by,

$$\sigma \sim \frac{1}{e^2}\sqrt{\frac{T^3}{m_e}}\frac{1}{\ln\Lambda} \tag{22}$$

For high temperatures, $T \gg m$,[6] (6) suggests that, $\ln\Lambda \sim 1$. Also, the mass of the plasma particles is dominated by thermal corrections, i.e. $m \sim T$, and $v \sim 1$. Consequently, in this case, (20) and (21) give for the conductivity,

$$\sigma \sim c\,\frac{T}{e^2} \tag{23}$$

[6] At not extremely high temperatures the roles of the electrons and the protons may be reversed [13] which would imply that $m = m_p$ (m_p being the proton mass) instead of m_e. This would decrease the conductivity by a factor of $(m_e/m_p)^{1/2} \simeq 0.02$. At even higher temperatures $T \gg m_p$, thermal corrections become dominant.

where c is a numerical factor of order unity.[7]

Using the above results we can estimate the diffusion length. Indeed, from (19), (22) and (23) we obtain,

$$l_d \sim \begin{cases} 10^8 GeV^{1/2} T^{-3/2} & T \geq 1\, MeV \\ \\ 10^8 GeV^{3/4} T^{-7/4} & T < 1\, MeV \end{cases} \tag{24}$$

An important point to stress is that the diffusion length is also increasing with time. If $l_d > \xi$ then the size of the correlated domains is actually determined by the diffusion length and it is the growth of the later that drives the evolution of the magnetic field configuration.

Another lengthscale necessary to consider is the magnetic Jeans length, given by [2] (see also [16] and [17]). This lengthscale is a measure of the dissipation of the field after t_{eq}. Indeed, in order for the field to have any astrophysical implications in structure formation, it needs to be coherent over,

$$\lambda_{_B}^{eq} \sim \frac{B^{eq}}{2\rho}\, m_{pl} \tag{25}$$

(i.e. $\xi_{eq} \geq \lambda_{_B}^{eq}$). If this is not the case then field oscillations dissipate the energy of the field and lead to its decay. If the field *is* coherent over the above lengthscale then it can have an accumulative effect and result in density instabilities, which can, by themselves, lead to structure and galaxy formation [1], [17].

4. Following the above analysis, the strength and coherence of the magnetic field configuration at any stage of its evolution[8], can be calculated if the intitial values of the field and the correlation length are given.

Being interested in the galactic magnetic fields, we would attempt to calculate the rms field on the scale of a protogalaxy $\sim 1\, Mpc$ at the time t_{eq}, when structure formation begins. To do so we need to employ a suitable averaging procedure.

Choosing the magnetic field as a stochastic variable, Enqvist and Olesen [7] have shown that the root mean square value of the field would behave as,

$$B_{rms} \equiv \sqrt{\langle B^2 \rangle} = \frac{1}{\sqrt{N}}\, B_{cd} \tag{26}$$

where N is the number of correlation lengthscales over which the field is averaged. At t_{eq} we have,

$$N = \frac{L_g^{eq}}{\xi_{eq}} \tag{27}$$

[7] As it is shown in [14] the main contribution to the early universe conductivity is from leptonic interactions. c is found to be a slowly increasing function of temperature (since at low temperatures most of the leptonic species heve underwent a pair–annihilation period). It is shown that c ranges from 0.07 at $T \sim 100\, MeV$ to 0.6 at $T \sim 100\, GeV$. For even higher temperatures c approaches the textbook estimate $c \simeq 1.3$ [15] for relativistic electron scattering off heavy ions.

[8] though not after t_{eq}

where L_g^{eq} is the protogalactic scale at t_{eq}, for which we find, $L_g^{eq} \sim (t_{eq}/t_{pr})^{2/3} L_g^{pr} \sim 10\, pc$, where $t_{pr} \sim 10^{18} sec$ is the present time and $L_g^{pr} = 1\, Mpc$.

At this point it should be mentioned that, in the above treatment, the rms value of the field has been computed as a line average. The averaging procedure could have an important effect on the results and has to be considered carefully. One argument in favour of line-averaging is that the observed galactic magnetic fields have been measured using the Faraday rotation of light spectra, which is also a line (line of sight) computation. If we assume that the ratio of the galactic dynamo seed field and the currently observed galactic field is independent of the averaging procedure then this would suggest line-averaging is required for the computation of the primordial rms field. However, the nonlinearity of the dynamo process as well as the rather poor knowledge we have for galaxy formation make such an assumption non-trivial. In any case, apart from the above, there seem to be no other argument in favour of a particular averaging procedure. Therefore, using line-averaging could be the safest choice.

Here it is also important to point out that *line averaging just gives an estimate of the rms field and does not correspond to any physical process.* Thus, it should not be confused with flux averaging which *does* correspond to a physical process, that of the field untangling, and is so in order to preserve flux conservation on scales larger than the diffusion length.

5. If the magnetic field is produced at very early times e.g. during an inflationary era, electroweak unification needs to be taken into account. If the original field is created before the electroweak transition then, assuming that it becomes "frozen" into the electroweak plasma is non–trivial. Indeed, during the electroweak era, since the electroweak symmetry group $SU(2) \times U(1)_Y$ is unbroken, there are four apparent "magnetic" fields, three of which are non-Abelian. It would be more precise, then, to refer only to the Abelian (hypercharge) part of the magnetic field, which satisfies the same magnetohydrodynamical equations as the Maxwell field of electromagnetism. The non-Abelian part of the field may not influence the motion of the plasma due to the existence of a temperature dependent magnetic mass, $m_B \approx 0.28 g^2 T$ [18], which could screen the field. Then, the motion of the plasma is determined primarily by the Abelian field and can reach a selfconsistent pattern, which will "lock" onto the field in the same way as in electromagnetism.

The condition for this screening to be effective is obtained by comparing the screening length $r_S \sim m_B^{-1}$ of the non-Abelian magnetic fields with the Larmor radius of the plasma motion $r_L \sim \frac{mv}{gB}$, where $m \sim \sqrt{\alpha}\, T$ ($\alpha = g^2/4\pi$) is the temperature induced physical mass of the plasma particles, g is the gauge coupling (charge) and v is the plasma particle velocity. Assuming thermal velocity distribution, i.e. $mv^2 \sim T$, suggests,

$$R \equiv \frac{r_L}{r_S} \sim 10^{-2} \frac{T^2}{B_{cd}} \tag{28}$$

If $R \geq 1$ then the restriction to the Abelian (Hypercharge) part of the magnetic field is justified. This restriction would not cause any significant change to the final magnitude of the "electromagnetic" magnetic field, since, at the electroweak transition, the hypercharge symmetry projects onto the photon through the Weinberg angle, $\cos\theta_W \approx 0.88$. If,

however, $R < 1$ then the non-Abelian fields do affect the plasma motion and should be taken into account. Since, $T \propto a^{-1}$ and B_{cd} falls at least as rapid as a^{-2}, R is in general an increasing function of time. Thus, the constraint has to be evaluated at the time of creation of the magnetic field configuration.

6. In determining the rms value of the field at t_{eq} one has also to take into account a number of constraints regarding its strength. An obvious requirement is that the magnetic field should not dominate the energy density of the universe. The expansion of the universe dilutes the energy density of the magnetic field, $\rho_B = B_{cd}^2/8\pi$, inside a correlated domain more effectively than the radiation density, which scales as a^{-4}. Therefore, it is sufficient to ensure that $\rho_B(t)$ is less than the energy density $\rho(t)$ of radiation at the time t_i of the formation of the magnetic field configuration. Thus, the constraint reads,

$$\rho_B(t_i) \leq \rho(t_i) \Rightarrow B_{cd}^i \leq \frac{\sqrt{3}}{2}\frac{m_{pl}}{t_i} \tag{29}$$

Another constraint comes from nucleosynthesis. This has been studied in detail by Cheng *et al.* [6]. They conclude that, at $t_{nuc} \sim 1\,sec$, the magnetic field should not be stronger than,

$$B^{nuc} \leq 10^{11} Gauss \tag{30}$$

on a scale larger than $\sim 10^4 cm$. A more recent treatment by Kernan *et al.* [16] relaxes the bound by about an order of magnitude, $B^{nuc} \leq e^{-1}(T_\nu^{nuc})^2 \sim 10^{12} Gauss$, where T_ν is the neutrino temperature and e is the electric charge. This bound is valid over all scales. Similar results are also reached by Grasso and Rubinstein [19].

Finally, an additional consideration would be the lower bound due to the galactic dynamo requirements, which, as already explained, demands a field of magnitude,

$$B^{eq} \geq 10^{-20} Gauss \tag{31}$$

over the comoving scale of $1\,Mpc$.

Our field evolution mechanism results in a more coherent magnetic field at t_{eq} than previously considered. As a result the rms field produced by the mechanisms in [5] could also be of greater strength. In some cases, the achieved rms magnetic field could be strong enough to dispense with the galactic dynamo.

At this point we should mention that there is some scepticism regarding the galactic dynamo [20], mostly due to the fact that, until now, there is no consistent dynamo model [21]. If there is no dynamo mechanism, the growth of the galactic magnetic field is only due to winding and, therefore, is linear (in contrast to exponential). Some authors believe that this linear amplification of the field would be enough to account for the currently observed galactic magnetic field, considering also the additional enchancement by line dragging during the gravitational collapse. It has been argued that, if this is the case, a seed field of the order of, $B^{eq} \sim 10^{-9} Gauss$ would be sufficient. The gravitational collapse would also sweep the intergalactic field lines into the galaxies leaving a relatively weak intergalactic

field, in agreement with observations, which find that the intergalactic field is less than $10^{-9} Gauss$.

7. To demonstrate the efficiency of the mechanism, in terms of magnitude and coherency, we will apply it to two toy-models of primordial magnetic field generation. For each one of these models the initial correlation length and the time of formation of the initial primordial magnetic field configuration is specified in such a way that it could correspond to a realistic situation. The initial magnitude of the generated magnetic field, though, is treated as a free parameter. In both cases we generate the field at a phase transition, when the universe is out of thermal equilibrium and, thus, the creation of such a field is acceptable [23].

CASE 1. *At the electroweak phase transition.*

In general it is thought that the electroweak transition is weakly first order, that is it occurs through bubble nucleation. A natural choice for the correlation length of a magnetic field, that is created at the transition, would be the bubbles' size at the time of their collision. Using typical values for the parameters, we find, $\xi_i \sim (v_w/\Gamma)^{1/4} \sim 10^{10} GeV^{-1} \sim 10^{-3} H_{ew}^{-1}$ [22], where $v_w \sim 0.1$ is the velocity of the bubble walls, $\Gamma \sim 10^{-41} GeV^4$ is the bubble nucleation rate per unit time per unit volume [24] and $H_{ew}^{-1} \sim t_{ew} \sim 10^{13} GeV^{-1}$ is the horizon size at the time of the transition. We assume that, by some mechanism, an initial magnetic field, $B_{cd}^i \sim 10^Z Gauss$ is generated at the transition, where Z is a free parameter. Using the above initial conditions we can explore the evolution of such a field.

From (11) we find that,

$$K \sim 10^{Z-4} GeV^{1/2} \tag{32}$$

Inserting the above into (12) we find,

$$v_A \sim 10^{Z-22} GeV^{-1/2} \frac{t^{1/2}}{\xi} \tag{33}$$

Using this in (14) we can estimate the correlation length ξ_{anh} at the time t_{anh} of pair annihilation,

$$\xi_{anh} \sim \begin{cases} 10^{15} GeV^{-1} & Z \leq 17 \\ \\ 10^{Z/2+7} GeV^{-1} & Z > 17 \end{cases} \tag{34}$$

The above suggests that the Alfven expansion dominates only for $Z > 17$. Otherwise it is the Hubble term that determines the evolution of the correlation length. Using (24) we can compute the diffusion length at t_{anh}. We find that $\xi_{anh} \gg l_d^{anh} \sim 10^{13} GeV^{-1}$, i.e. the correlation length is always larger than the diffusion length at that time.

From (12) and (15) it can be easily verified that, for all Z, $v_A(t_{anh}) > v_T(t_{anh})$. Thus, from the time of pair annihilation the correlation length evolves according to (17). Following its evolution likewise it can be shown that the Thomson expansion cannot compete with

the Hubble one. Thus, the evolution of the correlation length is driven by the expansion of the universe. At t_{eq} we find,

$$\xi_{eq} \sim \begin{cases} 10^{21} GeV^{-1} & Z \leq 17 \\ \\ 10^{Z/2+13} GeV^{-1} & Z > 17 \end{cases} \tag{35}$$

The diffusion length at t_{eq} is found by (24) to be, $l_d^{eq} \sim 10^{23} GeV^{-1}$. Thus, the above correlation length is larger than the diffusion length only if $Z > 20$. Therefore, the actual size of the correlated domains at t_{eq} is given by,

$$\xi_{eq} \sim \begin{cases} l_d^{eq} \sim 10^{23} GeV^{-1} & Z \leq 20 \\ \\ 10^{Z/2+13} GeV^{-1} & Z > 20 \end{cases} \tag{36}$$

With the use of the above and (11) we find,

$$B_{cd}^{eq} \sim \begin{cases} 10^{Z-25} Gauss & Z \leq 20 \\ \\ 10^{Z/2-15} Gauss & Z > 20 \end{cases} \tag{37}$$

Also, from (27) we have,

$$N \sim \begin{cases} 10^{10} & Z \leq 20 \\ \\ 10^{20-Z/2} & Z > 20 \end{cases} \tag{38}$$

Using the above (26) gives,

$$B_{rms}^{eq} \sim \begin{cases} 10^{Z-30} Gauss & Z \leq 20 \\ \\ 10^{3Z/4-25} Gauss & Z > 20 \end{cases} \tag{39}$$

By enforcing the various constraints we can specify the acceptable range of the final rms magnetic field strength and the corresponding necessary initial field.

The requirements of the galactic dynamo suggest that $Z \geq 10$. The energy density constraint demands that $Z \leq 26$ and the nucleosynthesis constraint requires $Z \leq 28$. Thus, the acceptable range of values for the magnetic field is,

$$\begin{array}{ccc} 10 & \leq Z \leq & 26 \\ \\ 10^{-20} Gauss & \leq B_{rms}^{eq} \leq & 10^{-6} Gauss \end{array} \tag{40}$$

Checking with the magnetic Jeans length we also find that the magnetic field could influence the structure formation process only if $Z < 13$. Finally, from (40) it is apparent that for some parameter space the galactic dynamo is not even required since an rms field

of strength up to a $\mu Gauss$ at t_{eq} can be achieved. In fact, we can dispense with the galactic dynamo for $Z > 21$.

CASE 2: *At grand unification.*

According to most scenaria for the breaking of grand unification, this occurs at a temperature $T_{GUT} \sim 10^{16} GeV$. Since not many details are specified for the nature of this phase transition the safest choice of a typical lengthscale is the horizon itself. Thus, for the initial correlation length we choose, $\xi_i \sim t_i \sim 10^{-15} GeV^{-1}$. Similarly with the electroweak treatment, we assume that, at the GUT transition[9] a magnetic field of strength, $B^i_{cd} \sim 10^W Gauss$ is generated, where W is treated as a free parameter.

In the same way as in the electroweak case we have,

$$K = 10^{W-43} GeV^{1/2} \tag{41}$$

with the use of which the correlation length at pair annihilation is,

$$\xi_{anh} \sim \begin{cases} l_d^{anh} \sim 10^{13} GeV^{-1} & W < 52 \\ \\ 10^{W/2-13} GeV^{-1} & W \geq 52 \end{cases} \tag{42}$$

Again, after pair annihilation it can be shown that $v_A(t_{anh}) > v_T(t_{anh})$. Thus the evolution of the correlation length continues with Thomson expansion. This time, however, the Thomson velocity is high enough to dominate the Hubble expansion rate. The resulting corrlation lengthscale at t_{eq} is,

$$\xi_{eq} \sim \begin{cases} l_d^{eq} \sim 10^{23} GeV^{-1} & W < 54 \\ \\ 10^{W/2-4} GeV^{-1} & W \geq 54 \end{cases} \tag{43}$$

Using the above we can calculate B^{eq}_{cd} and N in a similar way as in the electroweak case. Then it can immediately be shown that,

$$B^{eq}_{rms} \sim \begin{cases} 10^{W-69} Gauss & W < 54 \\ \\ 10^{3W/4-55} Gauss & W \geq 54 \end{cases} \tag{44}$$

By employing the various constraints on the above result we can determine the acceptable range of the magnetic field rms values. The galactic dynamo requirements suggest, $W \geq 49$, the energy density constraint demands, $W \leq 54$, the nucleosynthesis constraint requires, $W \leq 66$ and the non-Abelian constraint sets, $W \leq 50$. Thus, the acceptable range is very small, $49 \leq W \leq 50$ and corresponds to an rms magnetic field of order, $B^{eq}_{rms} \sim 10^{-(19-20)} Gauss$, which could just about seed the galactic dynamo mechanism. Of course, one can imagnine that, if the non-Abelian constraint is violated this should not necessarily mean that there will not be any surviving magnetic field. For instance, it is

[9] GUT here stands for Grand Unified Theory.

highly probable that, if the non-Abelian fields do influence the plasma motion they could perturb it in such a way that the magnetic field strength would diminish enough for the Larmor radious to increase over the non-Abelian screening length (with some of the magnetic energy being thermalized into the plasma). Thus, in this way stronger magnetic fields could survive the non-Abelian era. Moreover, such a mechanism could "cut-down" the initial strength of a primordial field and reduce it enough never to challenge the nucleosynthesis and energy density constraints. In that sense the non-Abelian stage of evolution of the field could expand the upper bound for W instead of reducing it to as low as 50. Still, it is difficult to imagnine a way of producing a stronger than $10^{-19} Gauss$ field, and in that sense the GUT-transition provides a narrow window towards a galactic seed field. By checking with the magnetic Jeans length, though, it can be shown that such a field would also influence the structure formation process and, therefore, it could have additional astrophysical effects.

The above was just a toy–model analysis of our mechanism. The mechanism can be applied to a variety of models. A more detailed and complicated example of such an application is given in [25], where a primordial magnetic field is created at the breaking of grand unification during inflation, in such a way that the narrow window of the GUT-case is achieved in a natural and realistic way.

8. In conclusion, we have analysed the evolution of primordial magnetic fields and shown that, when the effects of the surrounding plasma are taken into account, the correlation length of the field configuration grows faster than the Hubble expansion. This results into a more coherent magnetic field than previously thought. There is some similarity with the recent work of Brandenburg *et. al.* [26], who describe the plasma with relativistic magnetohydrodynamic equations and turbulent behaviour. However, they are unable to solve their equations analytically, and, instead, they attempt a numerical analysis in 2+1 dimensions. This also results in a faster growth of the coherence length. Still, they do not take into account all the effects of the plasma on the magnetic field configuration, such as, for example, the Thomson scattering effect.

We would like to thank K. Bajers, B. Carter, N. Turok, T. Vachaspati and especially M.J. Rees for dicussions. This work was partly supported by PPARC, the Greek State Scholarships Foundation (I.K.Y.) and the E.U. under the HCM program (CHRX-CT94-0423).

References

[1] I. Wasserman, Ap. J. **224** (1978) 337.

[2] P.J.E. Peebles, *The Large-Scale Structure of the Universe*, Princeton Series in Physics, Princeton Univ. Press, Princeton, New Jersey 1980.

[3] Y.B. Zeldovich, A.A. Ruzmaikin and D.D. Sokoloff, *Magnetic Fields in Astrophysics*, McGraw-Hill, New York 1983; A.A. Ruzmaikin, A.A. Shukurov and D.D. Sokoloff, *Magnetic Fields of Galaxies*, Kluwer, Dordrecht 1988;

[4] E.N. Parker, *Cosmical Magnetic Fields*, Oxford Univ. Press, Oxford 1979; H.K. Moffat, *Magnetic Field Generation in Electrically Conducting Fluids*, Cambridge Univ. Press, Cambridge 1978.

[5] M.S. Turner and L.M. Widrow, Phys. Rev. D **37** (1988) 2743; T. Vachaspati, Phys. Lett. B **265** (1991) 258; T. Vachaspati and A. Vilenkin, Phys. Rev. Lett. **67** (1991) 1057; B. Ratra, Ap. J. Lett. **391** (1992) L1; W.D. Garretson, G.B. Field and S.M. Carrol, Phys. Rev. D **46** (1992) 5346; A. Dolgov, Phys. Rev. D **48** (1993) 2499; K. Enqvist and P. Olesen, Phys. Lett. B **319** (1993) 178; K. Enqvist and P. Olesen, Phys. Lett. B **329** (1994) 195; B. Cheng and A. Olinto, Phys. Rev. D **50** (1994) 2421; M. Gasperini, M. Giovannini and G. Veneziano, Phys. Rev. Lett. **75** (1995) 3796.

[6] B. Cheng, D. Schramm and J.W. Truran, Phys. Rev. D **49** (1994) 5006.

[7] K. Enqvist, A.I. Rez and V.B. Semikoz, Nucl. Phys. B **436** (1995) 49.

[8] C.J. Hogan, Phys. Rev. Lett. **51** (1983) 1488.

[9] K.R. Lang, *Astrophysical Formulae*, Springer–Verlag, Berlin Heidelberg New York 1978.

[10] E.W. Kolb and M.S. Turner, *The Early Universe*, Frontiers in Physics, 1990.

[11] J.D. Jackson, *Classical Electrodynamics*, ed. by J. Wiley & Sons, New York London 1963.

[12] A.I. Akhiezer, I.A. Akhiezer, R.V. Polovin, A.G. Sitenko and K.N. Stepanov, *Plasma Electrodynamics*, Pergamon Press, Oxford 1975; N.A. Krall and A.W. Trivelpiece, *Principles of Plasma Physics*, McGraw-Hill New York 1973; G. Schmidt, *Physics of High Temperature Plasmas*, Academic Press, New York and London 1966.

[13] E.R. Harrison, Mon. Not. R. Astron. Soc. **147** (1970) 279.

[14] J. Ahonen and K. Enqvist, Phys. Lett. B **382** (1996) 40.

[15] E. Lifshitz and L.P. Pitaevskii, *Physical Kinetics*, Pergamon Press, Great Britain 1981.

[16] P.J. Kernan, G.D. Starkman and T. Vachaspati, preprint CWRU-P10-95 (astro-ph/9509126).

[17] E. Kim, A. Olinto and R. Rosner, preprint (astro-ph/9412070).

[18] A. Billoire, G. Lazarides and Q. Shafi, Phys. Lett. B **103** (1981) 450; T.A. DeGrand and D. Toussaint, Phys. Rev. D **25** (1982) 526; T.S. Biró and B. Müller, Nucl. Phys. A **561** (1993) 477; O. Philipsen; A.K. Rebhan, *Electroweak Physics and the Early Universe*, NATO ASI Series B: Physics Vol. 338 (pp. 393, 397), ed. by J.C. Romão and F. Freire, Plenum Press, New York and London 1994.

[19] D. Grasso and H.R. Rubinstein, Astropart. Phys. **3** (1995) 95; D. Grasso and H.R. Rubinstein, Phys. Lett. B **379** (1996) 73.

[20] Y. Sofue, M. Fujimoto and R. Wielebinski, Ann. Rev. Astr. Astrophys. **24** (1986) 459.

[21] S.I. Vainstein and R. Rosner, Ap. J. **376** (1991) 199; F. Cattaneo and S.I. Vainstein, Ap. J. Lett. **376** (1991) L29; S.I. Vainstein and F. Cattaneo, Ap. J. **393** (1992) 165; R.M. Kulsrud and S.W. Anderson, Ap. J. **396** (1992) 606.

[22] A.P. Martin and A.C Davis, Phys. Lett. B **360** (1995) 71.

[23] S. Davidson, Phys. Lett. B **380** (1996) 253.

[24] B.H. Liu, L. McLerran and N. Turok, Phys. Rev. D **46** (1992) 2668; N. Turok, Phys. Rev. Lett. **68** (1992) 1803; G. Baym, D. Bödeker and L. McLerran, Phys. Rev. D **53** (1996) 662.

[25] A.C. Davis and K. Dimopoulos, preprint CERN-TH/95-175, DAMTP-95-31 (astro-ph/9506132).

[26] A. Brandenburg, K. Enqvist and P. Olesen, Phys. Rev. D **54** (1996) 1291.